Quantum effects in one-photon and two-photon interference

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After introducing some basic definitions, the article describes several optical interference experiments in which quantum effects appear. An analysis of these experiments leads to some new and improved measurement techniques and to a better understanding of the quantum state. [S0034-6861(99)02102-9]

I. INTRODUCTION

Although interference is intrinsically a classical wave phenomenon, the superposition principle which underlies all interference is also at the heart of quantum mechanics. Feynman has referred to interference as really "the only mystery" of quantum mechanics. Furthermore, in some interference experiments we encounter the idea of quantum entanglement, which has also been described as really the only quantum mystery. Clearly interference confronts us with some quite basic questions of interpretation. Despite its long history, going back to Thomas Young at the beginning of the 19th century, optical interference still challenges our understanding, and the last word on the subject probably has not yet been written. With the development of experimental techniques for fast and sensitive measurements of light, it has become possible to carry out many of the Gedanken experiments whose interpretation was widely debated in the 1920s and 1930s in the course of the development of quantum mechanics. Although this article focuses entirely on experiments with light, interference has also been observed with many kinds of material particles like electrons, neutrons, and atoms. We particularly draw the reader's attention to the beautiful experiments with neutron beams by Rauch and co-workers and others (see, for example, Badurek *et al.*, 1988). Quantum optical interference effects are key topics of a recent book (Greenstein and Zajonc, 1997), an extended rather thorough review (Buzek and Knight, 1995) and an article in *Physics Today* (Greenberger *et al.*, 1993).

The essential feature of any optical interference experiment is that the light from several (not necessarily primary) sources like S_A and S_B (see Fig. 1) is allowed to come together and mix, and the resulting light intensity is measured at various positions. We characterize interference by the dependence of the resulting light intensities on the optical path length or phase shift, but we need to make a distinction between the measurement of a single realization of the optical field and the average over an ensemble of realizations or over a long time. A single realization may exhibit interference, whereas an ensemble average may not. We shall refer to the former as transient interference, because a single realization usually exists only for a short time. Transient interference effects have been observed in several optical experiments in the 1950s and 1960s. (Forrester *et al.*, 1955; Magyar and Mandel, 1963; Pfleegor and Mandel, 1967, 1968).

We now turn to interference effects that are defined in terms of an ensemble average. Let us start by distinguishing between second-order or one-photon, and fourth-order or two-photon interference experiments. In the simplest and most familiar type of experiment, one photodetector, say D_1, is used repeatedly to measure the probability $P_1(x_1)$ of detecting a photon in some short time interval as a function of position x_1 [see Fig. 1(a)]. Interference is characterized by the (often, but not necessarily, periodic) dependence of $P_1(x_1)$ on the optical path lengths S_AD_1 and S_BD_1 or on the corresponding phase shifts ϕ_{A1} and ϕ_{B1}. Because $P_1(x_1)$ depends on the second power of the optical field and on the detection of one photon at a time, we refer to this as second-order, or one-photon, interference. Sometimes two photodetectors D_1 and D_2 located at x_1 and x_2 are used in coincidence repeatedly to measure the joint probability $P_2(x_1,x_2)$ of detecting one photon at x_1 and one at x_2 within a short time [see Fig. 1(b)]. Because $P_2(x_1,x_2)$ depends on the fourth power of the field, we refer to this as fourth-order, or two-photon, interference. For the purpose of this article, a photon is any eigenstate of the total number operator belonging to the eigenvalue 1. That means that a photon can be in the form of an infinite plane wave or a strongly localized wave packet. Because most photodetectors function by photon absorption, the appropriate dynamical variable

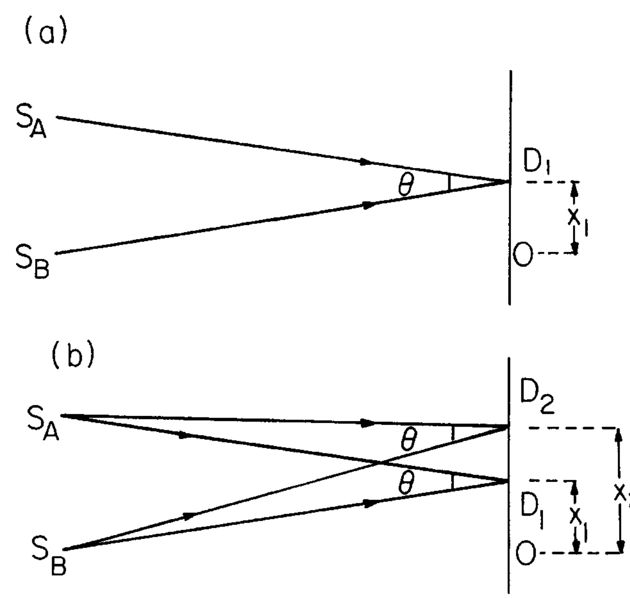

FIG. 1. Principle of photon interference: (a) one-photon or second-order interference; (b) two-photon or fourth-order interference. S_A and S_B are sources. D_1 and D_2 are photodetectors.

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for describing the measurement is the photon annihilation operator. If we make a Fourier decomposition of the total-field operator $\hat{E}(x)$ at the detector into its positive- and negative-frequency parts $\hat{E}^{(+)}(x)$ and $\hat{E}^{(-)}(x)$, then these play the roles of photon annihilation and creation operators in configuration space. Let $\hat{E}^{(+)}(x_1)$, $\hat{E}^{(+)}(x_2)$ be the positive-frequency parts of the optical field at the two detectors. Then $P_1(x_1)$ and $P_2(x_1,x_2)$ are given by the expectations in normal order:

$$P_1(x_1)=\alpha_1\langle\hat{E}^{(-)}(x_1)\hat{E}^{(+)}(x_1)\rangle, \tag{1}$$

$$P_2(x_1,x_2)=\alpha_1\alpha_2\langle\hat{E}^{(-)}(x_1)\hat{E}^{(-)}(x_2)\hat{E}^{(+)}(x_2)\hat{E}^{(+)}(x_1)\rangle, \tag{2}$$

where α_1, α_2 are constants characteristic of the detectors and the measurement times.

II. SECOND-ORDER INTERFERENCE

Let us decompose $\hat{E}^{(+)}(x_1)$ and $\hat{E}^{(+)}(x_2)$ into two normal modes A and B, such that $\hat{a}_A,\hat{a}_B$ are the annihilation operators for the fields produced by the two sources S_A and S_B, respectively. Then we may put $\hat{E}^{(+)}(x_1)=f_Ae^{i\phi A1}\hat{a}_A+f_Be^{i\phi B1}\hat{a}_B$, where f_A, f_B are complex parameters, and similarly for $\hat{E}^{(+)}(x_2)$. From Eq. (1) we then find

$$P_1(x_1)=\alpha_1[|f_A|^2\langle\hat{n}_A\rangle+|f_B|^2\langle\hat{n}_B\rangle +f_A^*f_Be^{i(\phi_{B1}-\phi_{A1})}\langle\hat{a}_A^\dagger\hat{a}_B\rangle+\text{c.c.}]. \tag{3}$$

If second-order interference is characterized by the dependence of $P_1(x_1)$ on the optical path lengths or on the phase difference $\phi_{B1}-\phi_{A1}$, then clearly the condition for the system to exhibit second-order interference is that $\langle\hat{a}_A^\dagger\hat{a}_B\rangle\neq 0$. This is usually achieved most easily if the fields from the two sources S_A and S_B are at least partly correlated or mutually coherent. We define the degree of second-order mutual coherence by the normalized correlation ratio

$$|\gamma_{AB}^{(1,1)}|\equiv|\langle\hat{a}_A^\dagger\hat{a}_B\rangle|/(\langle\hat{a}_A^\dagger\hat{a}_A\rangle\langle\hat{a}_B^\dagger\hat{a}_B\rangle)^{1/2}, \tag{4}$$

so that, by definition, $|\gamma_{AB}^{(1,1)}|$ lies between 0 and 1. But such correlation is not necessary for interference. Even with two independent sources it is apparent from Eq. (3) that interference can occur if $\langle\hat{a}_A\rangle\neq 0\neq\langle\hat{a}_B\rangle$. An example would be the two-mode coherent state $|v_A\rangle_A|v_B\rangle_B$, with complex eigenvalues v_A,v_B, for which $\langle\hat{a}_A^\dagger\hat{a}_B\rangle=v_A^*v_B$, which is nonzero because of the definite complex amplitude of the field in a coherent state. The field of a single-mode laser is often well approximated by a coherent state for a short time. On the other hand the corresponding expectations vanish for a field in a Fock (photon number) state $|n_A\rangle_A|n_B\rangle_B$, for which $\langle\hat{a}_A^\dagger\hat{a}_B\rangle=0$. Therefore there is no second-order interference in this case. Needless to say, this situation has no obvious counterpart in classical optics.

In order to understand why interference effects occur in some cases and not in others, we need to recall that interference is the physical manifestation of the intrinsic indistinguishability of the sources or of the photon paths. If the different possible photon paths from source to detector are indistinguishable, then we have to add the corresponding probability amplitudes before squaring to obtain the probability. This results in interference terms as in Eq. (3). On the other hand, if there is some way, even in principle, of distinguishing between the possible photon paths, then the corresponding probabilities have to be added and there is no interference.

Let us see how this argument works when each source consists of a single two-level atom. When the atom is in the fully excited state (an energy eigenstate), its energy can be measured, in principle, without disturbing the atom. Suppose that both sources are initially in the fully excited state and that the energy of each atom is measured immediately after the detection of a photon by D_1. If source A is found to be in the ground state whereas source B is found to be still excited, then, obviously, S_A can be identified as the source of the photon detected by D_1. Therefore there is no second-order interference in this case, and this conclusion holds regardless of whether the energy measurement is actually carried out. In this case, the optical field is in a one-photon Fock state $|1\rangle_A|0\rangle_B$ for which $\langle\hat{a}_A^\dagger\hat{a}_B\rangle=0$. On the other hand, if the atoms are in a superposition of upper and lower states initially, then the atomic energy has no well-defined initial value and it cannot be measured without disturbing the atom. The source of the detected photon therefore cannot be identified by measuring the atomic energy, or in any other way, and, as a result, second-order interference is observed. This argument can be made more quantitative in that the degree of second-order coherence $|\gamma_{AB}^{(1,1)}|$ in Eq. (4) can be shown to equal the degree of path indistinguishability (Mandel, 1991).

It should be clear from the foregoing that in these experiments one photon does not interfere with another one; only the two probability amplitudes of the same photon interfere with each other. This has been confirmed more explicitly in interference experiments with a single photon (Grangier *et al.*, 1986) and in experiments with two independent laser beams, in which interference was observed even when the light was so weak that one photon passed through the interferometer and was absorbed by the detector long before the next photon came along (Pfleegor and Mandel, 1967, 1968).

III. FOURTH-ORDER INTERFERENCE

We now turn to the situation illustrated in Fig. 1(b), in which two photodetectors are used in coincidence to measure the joint probability $P_2(x_1,x_2)$ of detecting one photon at x_1 and one at x_2. Fourth-order interference occurs when $P_2(x_1,x_2)$ depends on the phase differences $\phi_{A1}-\phi_{B2}$, and this happens when the different paths of the photon pair from the sources to the detectors are indistinguishable. Then we again have to add the corresponding (this time two-photon) probability amplitudes before squaring to obtain the probability. From Eq. (2) one can show that

$$P_2(x_1,x_2)=\alpha_1\alpha_2\{|f_A|^4\langle{:}\hat{n}_A^2{:}\rangle+|f_B|^4\langle{:}\hat{n}_B^2{:}\rangle+2|f_A|^2|f_B|^2\langle\hat{n}_A\rangle\langle\hat{n}_B\rangle[1+\cos(\phi_{B2}-\phi_{A2}+\phi_{A1}-\phi_{B1})]$$
$$+f_A^{*2}f_B^2\langle\hat{a}_A^{\dagger 2}\hat{a}_B^2\rangle e^{i(\phi_{B2}-\phi_{A2}+\phi_{B1}-\phi_{A1})}+\text{c.c.}+|f_A|^2f_A^*f_B\langle\hat{a}_A^{\dagger 2}\hat{a}_A\hat{a}_B\rangle[e^{i(\phi_{B1}-\phi_{A1})}+e^{i(\phi_{B2}-\phi_{A2})}]+\text{c.c.}$$
$$+|f_B|^2f_B^*f_A\langle\hat{a}_B^{\dagger 2}\hat{a}_B\hat{a}_A\rangle[e^{i(\phi_{A1}-\phi_{B1})}+e^{i(\phi_{A2}-\phi_{B2})}]+\text{c.c.}\}, \quad (5)$$

where $\langle{:}\hat{n}^r{:}\rangle$ denotes the rth normally ordered moment of $\hat{n}$.

For illustration, let us focus once again on the special case in which each source consists of a single excited two-level atom. We have seen that in this case there is no second-order interference, because the source of each detected photon is identifiable in principle. But the same is not true for fourth-order interference of the photon pair. This time there are two indistinguishable two-photon paths, viz., (a) the photon from S_A is detected by D_1 and the photon from S_B is detected by D_2 and (b) the photon from S_A is detected by D_2 and the photon from S_B is detected by D_1. Because cases (a) and (b) are indistinguishable, we have to add the corresponding two-photon amplitudes before squaring to obtain the probability, and this generates interference terms. In this case most terms on the right of Eq. (5) vanish, and we immediately find the result given by (see Box A)

Box A: Comparison of quantum and classical fourth-order interference.

For the case in which exactly one photon is emitted by S_A and one photon by S_B, we have

$\langle{:}\hat{n}_A^2{:}\rangle=\langle\hat{n}_A(\hat{n}_A-1)\rangle=0,\quad \langle{:}\hat{n}_B^2{:}\rangle=\langle\hat{n}_B(\hat{n}_B-1)\rangle=0,$

$\langle\hat{a}_A^{\dagger 2}\hat{a}_B^2\rangle=0,\quad \langle\hat{a}_A^{\dagger 2}\hat{a}_A\hat{a}_B\rangle=0,\quad \langle\hat{a}_B^{\dagger 2}\hat{a}_B\hat{a}_A\rangle=0.$

Therefore of all the terms on the right hand side of Eq. (5) only the third survives, and we obtain finally,

$$P_2(x_1,x_2)=\alpha_1\alpha_2 2|f_A|^2|f_B|^2[1+\cos(\phi_{B2}-\phi_{A2}+\phi_{A1}-\phi_{B1})]. \quad (6)$$

This result exhibits two-photon interference with 100% visibility.

Let us contrast this conclusion for a two-photon state with the result given by the same Eq. (5) for a classical state of the incoming field, when the two sources are completely independent. In this case we have to treat $\hat{a}_A,\hat{a}_B$ as complex c-number amplitudes, $\langle\hat{n}_A\rangle$ becomes the mean light intensity $\langle I_A\rangle$, and $\langle{:}\hat{n}_A^2{:}\rangle$ becomes $\langle I_A^2\rangle$. Because of the phase independence, all terms on the right hand side of Eq. (5) beyond the first three vanish again, but this time terms one and two are nonzero, and we have the result

$$P_2(x_1,x_2)=\alpha_1\alpha_2[\langle(|f_2|^2I_A+|f_B|^2I_B)^2\rangle+2|f_A|^2|f_B|^2\langle I_A\rangle\langle I_B\rangle\cos(\phi_{B2}-\phi_{A2}+\phi_{A1}-\phi_{B1})].$$

It is not difficult to prove that in this classical field case the visibility of the interference has an upper bound of 1/2, compared with the value 1 given by Eq. (6) for the case of a quantum field (Richter, 1979).

$$P_2(x_1,x_2)=\alpha_1\alpha_2 2|f_A|^2|f_B|^2\times[1+\cos(\phi_{B2}-\phi_{A2}+\phi_{A1}-\phi_{B1})]. \quad (6)$$

Despite the fact that the two sources are independent, they exhibit two-photon interference with 100 percent visibility.

Two-photon interference exhibits some striking nonlocal features. For example, $P_2(x_1,x_2)$ given by Eq. (6) can be shown to violate one or more of the Bell inequalities that a system obeying local realism must satisfy. This violation of locality, which is discussed more fully in the article by Zeilinger in this issue, has been demonstrated experimentally. [See, for example, Mandel and Wolf, 1995.]

IV. INTERFERENCE EXPERIMENTS WITH A PARAMETRIC DOWNCONVERTER SOURCE

The first two-photon interference experiment of the type illustrated in Fig. 1(b), in which each source delivers exactly one photon simultaneously, was probably the one reported by Ghosh and Mandel in 1987. They made use of the signal and idler photons emitted in the splitting of a pump photon in the process of spontaneous parametric downconversion in a nonlinear crystal of $LiIO_3$. The crystal was optically pumped by the 351.1-nm uv beam from an argon-ion laser and from time to time it gave rise to two simultaneous signal and idler photons at wavelengths near 700 nm. A modified and slightly improved version of the experiment was later described by Ou and Mandel (1989). The signal (s)

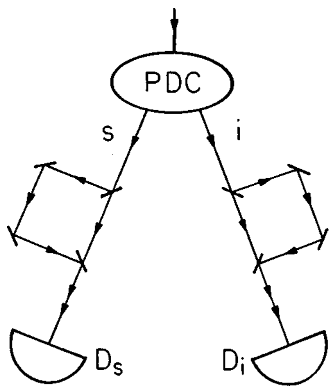

FIG. 2. Principle of the Franson (1989) two-photon interference experiment in which signal and idler photons never mix. PDC is the parametric downconverter. D_s and D_i are photodetectors.

and idler (i) photons were incident from opposite sides on a 50%:50% beam splitter that mixed them at a small angle $\theta \approx 1$ mrad, and the two mixed beams then fell on detectors D_1 and D_2, each of which carried a 0.1-mm-wide aperture. The photons counted by each detector separately and by the two detectors in coincidence in a total time of a few minutes were registered for various positions of the detectors. Because of the two-photon state, no second-order interference is expected from quantum mechanics, as we have seen, and none was observed. However, the two-photon coincidence rate exhibited the expected interference in the form of a periodic variation of the rate with detector position, because the photon pair detected by D_1 and D_2 could have originated as signal and idler, respectively, or vice versa.

An ingenious variation on the same theme of two-photon interference was proposed by Franson (1989) and is illustrated in Fig. 2. Signal and idler photons emitted simultaneously in two slightly different directions from a parametric downconverter (PDC) fall on two detectors D_s and D_i, respectively. The two beams never mix. On the way to the detector each photon encounters a beam splitter leading to an alternative time-delayed path, as shown, and each photon is free to follow either the shorter direct or the longer delayed path. If the time difference T_D between the long and short paths is much longer than the coherence time T_C of the downconverted light, and much longer than the coincidence resolving time T_R, no second-order interference is to be expected, and at first glance it might seem that no fourth-order interference would occur either. But the signal and idler photons are emitted simultaneously, and, within the coincidence resolving time, they are detected simultaneously. Therefore in every coincidence both photons must have followed the short path or both photons must have followed the long path, but we cannot tell which. When $T_C \ll T_D \ll T_R$ two more path combinations are possible. With continuous pumping of the parametric downconverter the emission time is random and unknown, and there is no way to distinguish between the light paths. We therefore have to add the corresponding probability amplitudes, which leads to the

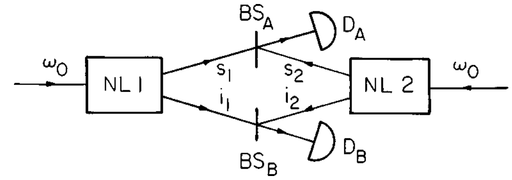

FIG. 3. Principle of the interference experiment with two downconverters in which both one-photon and two-photon interference can be investigated (after Ou *et al.*, 1990).

prediction of fourth-order interference as the path difference in one arm is varied. This has been confirmed experimentally. A different outcome may be encountered with pulsed rather than continuous excitation of the parametric downconverter.

V. INTERFERENCE EXPERIMENTS WITH TWO PARAMETRIC DOWNCONVERTERS

Next let us consider the experiment illustrated in Fig. 3, which allows both one-photon and two-photon interference to be investigated at the same time. Two similar nonlinear crystals NL1 and NL2, which both function as parametric downconverters, are optically pumped simultaneously by mutually coherent pump beams that we shall treat classically and represent by the complex field amplitudes V_1 and V_2. As a result downconversion can occur at NL1, with the simultaneous emission of a signal s_1 and an idler i_1 photon in two slightly different directions, or downconversion can occur at NL2, with the simultaneous emission of an s_2 and an i_2 photon, as shown. The question we wish to address is whether, in view of the mutual coherence of the two pump beams, the s_1 and s_2 beams from the two downconverters are mutually coherent and exhibit interference when they are mixed, and similarly for the i_1 and i_2 beams. In order to answer the question the experiment illustrated in Fig. 3 is carried out. s_1 and s_2 are allowed to come together; they are mixed at the 50%:50% signal beam splitter BS_A, and the combined beam emerging from BS_A falls on the photon detector D_A. If s_1 and s_2 are mutually coherent, then the photon counting rate of D_A varies sinusoidally as the phase difference between the two pump beams V_1 and V_2 is slowly increased. Similarly for the two idlers i_1 and i_2, which are mixed by BS_B and detected by D_B.

In order to treat this problem theoretically we represent the quantum state of the signal and idler photon pair from each crystal by the entangled state $|\Psi_j\rangle = M_j|\mathrm{vac}\rangle_{s_j,i_j} + \eta V_j|1\rangle_{s_j}|1\rangle_{i_j}$ $(j=1,2)$. The combined state is then the direct product state $|\Psi\rangle = |\Psi_1\rangle \times |\Psi_2\rangle$, because the two downconversions proceed independently. V_1 and V_2 are the c-number complex amplitudes of the pump fields. η represents the coupling between pump modes and the downconverted signal and idler modes, such that $\langle|\eta V_j|^2\rangle$ $(j=1,2)$ is the small probability of downconversion in a short measurement time. M_1 and M_2 are numerical coefficients that ensure the normalization of $|\Psi_1\rangle$ and $|\Psi_2\rangle$, which we take to be real

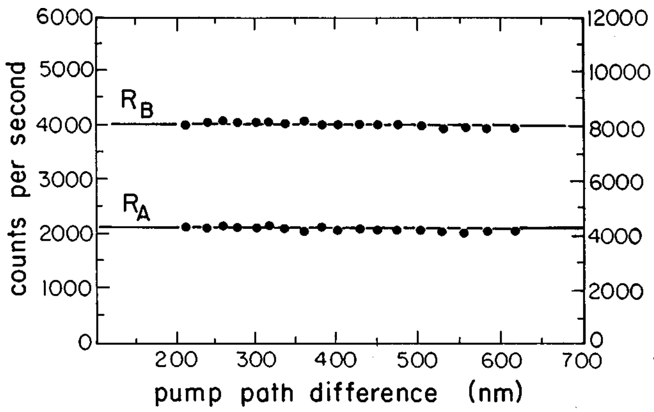

FIG. 4. Results of measurements of the photon counting rate by D_A and D_B in Fig. 3 as a function of path difference, showing the absence of one-photon interference.

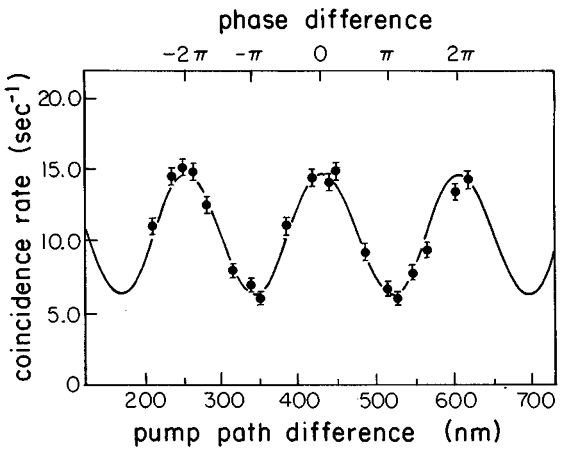

FIG. 5. Results of coincidence measurements by D_A and D_B in Fig. 3 as a function of path difference, showing two-photon interference. The continuous curve is theoretical.

for simplicity. Because $\langle|\eta V_j|^2\rangle \ll 1$ it follows that M_1 and M_2 are very close to unity. We shall retain the coefficients M_1 and M_2 nevertheless, because they provide us with useful insight into the role played by the vacuum. Of course the downconverted light usually has a very large bandwidth, and treating each signal and idler as occupying one monochromatic mode is a gross oversimplification. However, a more exact multimode treatment leads to very similar conclusions about the interference.

The positive-frequency parts of the signal and idler fields at the two detectors can be given the two-mode expansions $\hat{E}_s^{(+)}=\hat{a}_{s1}e^{i\theta_{s1}}+i\hat{a}_{s2}e^{i\theta_{s2}}$ and $\hat{E}_i^{(+)}=\hat{a}_{i1}e^{i\theta_{i1}}+i\hat{a}_{i2}e^{i\theta_{i2}}$, where $\theta_{s1},\theta_{s2},\theta_{i1},\theta_{i2}$ are phase shifts corresponding to the propagation from one of the two sources NL1, NL2 to one of the two detectors D_A, D_B. Then the expectations of the number of photons detected by D_A and by D_B are $\langle\Psi|\hat{E}_s^{(-)}\hat{E}_s^{(+)}|\Psi\rangle$ and $\langle\Psi|\hat{E}_i^{(-)}\hat{E}_i^{(+)}|\psi\rangle$, and for the quantum state $|\Psi\rangle=|\Psi_1\rangle\times|\Psi_2\rangle$ we obtain immediately

$$\langle\Psi|\hat{E}_s^{(-)}\hat{E}_s^{(+)}|\Psi\rangle=|\eta|^2(\langle|V_1|^2\rangle+\langle|V_2|^2\rangle)$$
$$=\langle\Psi|\hat{E}_i^{(-)}\hat{E}_i^{(+)}|\Psi\rangle. \quad (7)$$

These averages are independent of the interferometric path lengths and of the phases of the two pump beams, showing that there is no interference and no mutual coherence between the two signals s_1, s_2 or between the two idlers i_1, i_2. These conclusions are confirmed by the experimental results presented in Fig. 4, which exhibit no sign of second-order or one-photon interference.

Next let us look at the possibility of fourth-order or two-photon interference, by measuring the joint probability of detecting a signal photon and an idler photon with both detectors in coincidence. This probability is proportional to $P_{12}=\langle\Psi|\hat{E}_s^{(-)}\hat{E}_i^{(-)}\hat{E}_i^{(+)}\hat{E}_s^{(+)}|\Psi\rangle$, and it is readily evaluated. If $|V_1|^2=I=|V_2|^2$ and $|\eta|^2 I \ll 1$, so that terms of order $|\eta|^4 I^2$ can be neglected, we find

$$P_{12}=2|\eta|^2\langle I\rangle[1-M_1M_2|\gamma_{12}^{(1,1)}|\cos\Theta], \quad (8)$$

where $\Theta\equiv\theta_{s2}+\theta_{i2}-\theta_{s1}-\theta_{i1}+\arg(\gamma_{12}^{(1,1)})$ and $\gamma_{12}^{(1,1)} \equiv\langle V_1^* V_2\rangle/\langle I\rangle$ is the complex degree of coherence of the two classical pump beams. A two-photon coincidence measurement with both detectors D_A and D_B is therefore expected to exhibit interference as the optical path difference or the pump phase difference is varied. This is confirmed by the experimental results shown in Fig. 5. It is interesting to note that the vacuum contribution to the state plays an essential role, because of the presence of the M_1M_2 coefficients in Eq. (8).

Finally, we would like to understand in physical terms why no second-order interference is registered by detectors D_A and D_B separately, but fourth-order interference is registered by the two together. Here it is helpful to recall the relationship between interference and indistinguishability. From the coincidence measurement in Fig. 5 it is impossible to determine whether the detected photon pair originates in NL1 or in NL2, and this indistinguishability is manifest as a fourth-order interference pattern. However, if we are interested only in the interference of, say, the signal photons registered by D_A, we can use the detection of the idlers as an auxiliary source of information, to determine where each detected signal photon originated. This destroys the indistinguishability of the two sources and kills the interference of the signal photons, whether or not the auxiliary measurement is actually carried out.

Figure 6 illustrates a one-photon interference experiment with two downconverters that exhibits interesting nonclassical features (Zou *et al.*, 1991). NL1 and NL2 are two similar nonlinear crystals of $LiIO_3$ functioning as parametric downconverters. They are both optically pumped by the mutually coherent uv light beams from an argon-ion laser oscillating on the 351.1-nm spectral line. As a result, downconversion can occur at NL1 with the simultaneous emission of a signal s_1 and an idler i_1 photon at wavelengths near 700 nm, or it can occur at NL2 with the simultaneous emission of an s_2 and i_2 photon. Simultaneous downconversions at both crystals is very improbable. NL1 and NL2 are aligned so as to make i_1 and i_2 collinear and overlapping, as shown, so that a photon detected in the i_2 beam could have come

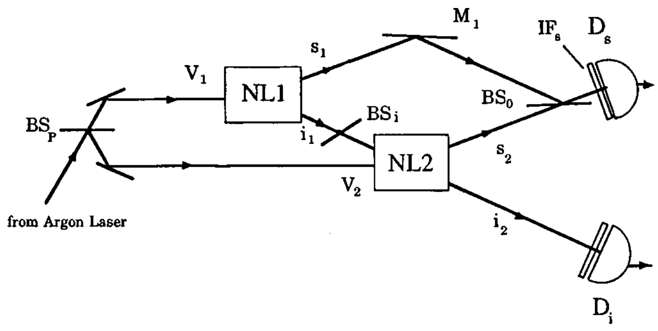

FIG. 6. Outline of the one-photon interference experiment with two downconverters (Zou *et al.*, 1991). See text for description.

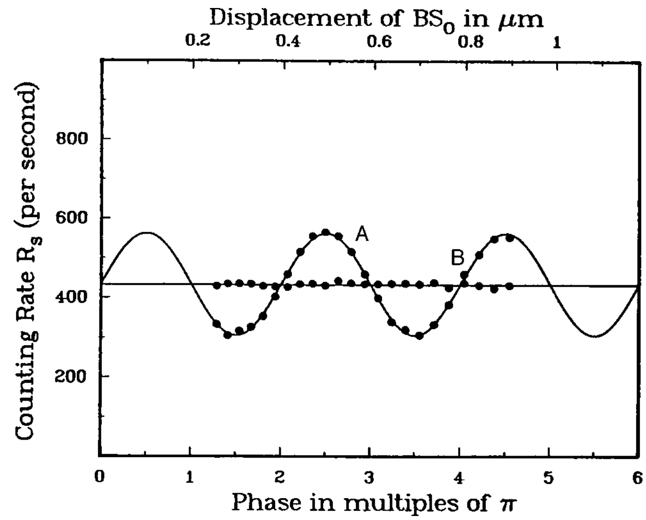

FIG. 7. Results of the one-photon interference experiment shown in Fig. 6: *A*, data with i_1 unblocked; *B*, data with i_1 blocked.

from NL1 or NL2. At the same time the s_1 and s_2 signal beams come together and are mixed at beam splitter BS_0. The question to be explored is whether, in view of the mutual coherence of the two pump beams, s_1 and s_2 are also mutually coherent and exhibit interference, under the conditions when the downconversions at NL1 and NL2 are spontaneous and random. More explicitly, if BS_0 is translated in a direction normal to its face. will the photon counting rate of detector D_s vary sinusoidally, thereby indicating that interference fringes are passing across the photocathode?

With the experiment in Fig. 3 in mind, one might not expect to see one-photon interference at D_s, but, as shown in Fig. 7 (curve *A*), interference fringes were actually observed so long as i_1 and i_2 were well aligned and overlapped. The relatively small visibility of the interference is largely due to the incomplete overlap of the two idlers. However, after deliberate misalignment of i_1 and i_2, or if i_1 was blocked from reaching NL2, all interference disappeared, as shown by curve *B* in Fig. 7. Yet the average rate of photon emission from NL2 was unaffected by blocking i_1 or by misalignment. In the absence of induced emission from NL2, how can this be understood?

Here it is instructive again to invoke the relationship between interference and indistinguishability. Let us suppose that an auxiliary perfect photodetector D_i is placed in the path of the i_2 beam equidistant with D_s from NL2, as shown in Fig. 6. Now the insertion of D_i in the path of i_2 does not in any way disturb the interference experiment involving the s_1 and s_2 beams. However, when i_1 is blocked, D_i provides information about the source of the signal photon detected by D_s. For example, if the detection of a signal photon by D_s is accompanied by the simultaneous detection of an idler photon by D_i, a glance at Fig. 6 shows immediately that the signal photon (and the idler) must have come from NL2. On the other hand, if the detection of a signal photon by D_s is not accompanied by the simultaneous detection of an idler by D_i, then the signal photon cannot have come from NL2 and must have originated in NL1. With the help of the auxiliary detector D_i we can therefore identify the source of each detected signal photon, whenever i_1 is blocked, and this distinguishability wipes out all interference between s_1 and s_2. A similar conclusion applies when the two idlers i_1 and i_2 do not overlap, so that they can be measured separately. However, when i_1 is unblocked and the two idlers overlap, this source identification is no longer possible, and s_1 and s_2 exhibit interference. Needless to say, it is not necessary actually to carry out the auxiliary measurement with D_i; the mere possibility, in principle, that such a measurement could determine the source of the signal photon is sufficient to kill the interference of s_1 and s_2.

This kind of argument leads to an important conclusion about the quantum state of a system: in an experiment the state reflects not what is actually known about the system, but rather what is knowable, in principle, with the help of auxiliary measurements that do not disturb the original experiment. By focusing on what is knowable in principle, and treating what is known as largely irrelevant, one completely avoids the anthropomorphism and any reference to consciousness that some physicists have tried to inject into quantum mechanics. We emphasize here that the act of blocking the path of i_1 between NL1 and NL2 kills the interference between s_1 and s_2 not because it introduces a large uncontrollable disturbance. After all, the signal photons s_1 and s_2 are emitted spontaneously and the spontaneous emissions are not really disturbed at all by the act of blocking i_1. In this experiment the disturbance introduced by blocking i_1 is of a more subtle kind: it is only the possibility of obtaining information about the source of the signal photon which is disturbed by blocking i_1.

If, instead of blocking i_1 completely from reaching NL2, one merely attenuates i_1 with some sort of optical filter of complex transmissivity $\mathcal{T}$, then the degree of coherence and the visibility of the interference pattern formed by s_1 and s_2 are reduced by the factor $|\mathcal{T}|$ (Zou *et al.*, 1991). This provides us with a convenient means for controlling the degree of coherence of two light-

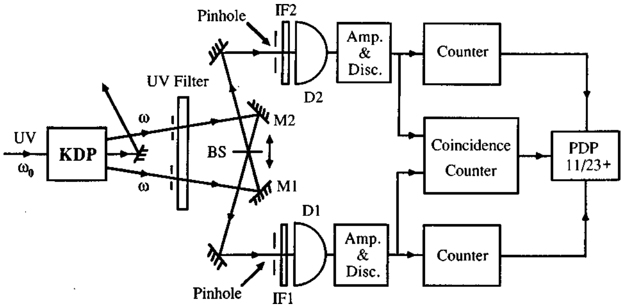

FIG. 8. Outline of the two-photon interference experiment to measure the time separation between signal and idler photons (Hong *et al.*, 1987). See text for description.

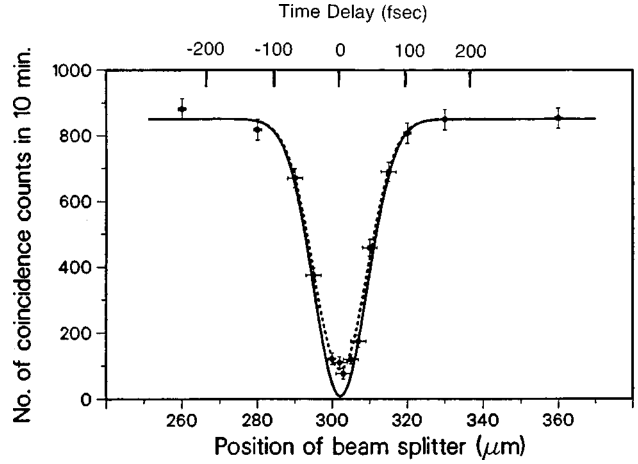

FIG. 9. Results of the two-photon interference experiment shown in Fig. 8. The measured coincidence rate is plotted as a function of beam-splitter displacement in μm or differential time delay in fsec. The continuous curve is theoretical.

beams s_1 and s_2 with a variable filter acting on i_1, without affecting the light intensities of s_1 and s_2. Finally, insofaras i_1 falling on NL2 may be said to induce coherence between s_1 and s_2 from the two sources NL1 and NL2, we have here an example of induced coherence without induced emission.

VI. MEASUREMENT OF THE TIME INTERVAL BETWEEN TWO PHOTONS BY INTERFERENCE

The same fourth-order two-photon interference effect has been used to measure the time separation between two photons with time resolution millions of times shorter than the resolution of the detectors and the electronics (Hong *et al.*, 1987). Let us consider the experiment illustrated in Fig. 8. Here the signal and idler photons emitted from a uv-pumped crystal of potassium dihydrogen phosphate (KDP) serving as parametric downconverter are sent in opposite directions through a symmetric 50%:50% beam splitter (BS) that mixes them. The emerging photon pair is allowed to impinge on two similar photon detectors D_1 and D_2, whose output pulses are counted both separately and in coincidence as the beam splitter is translated in the direction shown though a distance of a few wavelengths. The coherence time T_c of the downconverted light is made about 10^{-13} sec with the help of the interference filters IF_1, IF_2.

Let us consider the quantum state $|\psi\rangle$ of the photon pair emerging from the beam splitter. With two photons impinging on BS from opposite sides there are really only three possibilities for the light leaving BS: (a) one photon emerges from each of the outputs 1 and 2; (b) two photons emerge from output 1 and none emerges from output 2; (c) two photons emerge from output 2 and none emerges from output 1. The quantum state of the beam-splitter output is actually a linear superposition of all three possibilities in the form

$$|\psi\rangle=(|\mathcal{R}|^2-|\mathcal{T}|^2)|1\rangle_1|1\rangle_2 + \sqrt{2}i|\mathcal{R}\mathcal{T}|[|2\rangle_1|0\rangle_2+|0\rangle_1|2\rangle_2], \tag{9}$$

where $\mathcal{R}$ and $\mathcal{T}$ are the complex beam-splitter reflectivity and transmissivity. When $|\mathcal{R}|=1/\sqrt{2}=|\mathcal{T}|$, the first term on the right vanishes, which implies the destructive interference of the photon pair in arms 1 and 2, and both photons emerge together either in arm 1 or in arm 2. Therefore no coincidence counts (other than accidentals) between detectors D_1 and D_2 are registered. The reason for this can be understood by reference to Fig. 8. A coincidence detection between D_1 and D_2 can occur only if the two incoming signal and idler photons are either both reflected from the beam splitter or are both transmitted through the beam splitter. Because these two possible photon paths are indistinguishable, we have to add the corresponding two-photon probability amplitudes before squaring to obtain the probability. But because of the phase change that occurs on reflection from the beam splitter, as compared with that on transmission, the two-photon probability amplitude for two reflections from BS is 180° out of phase with the corresponding two-photon probability amplitude for two transmissions through BS. When these two amplitudes are added they give zero.

Needless to say, this perfect destructive interference of the photon pair requires two identical incident photons, and their description goes well beyond our oversimplified two-mode treatment. If we think of the incoming entangled photon pair as two identical wave packets that overlap completely in time, then it should be obvious that if one wave packet is delayed even slightly relative to the other, perfect destructive interference is no longer possible, and the apparatus in Fig. 8 no longer yields zero coincidences. The greater the relative time delay τ_D, the greater is the two-photon coincidence rate R_c, and by the time the delay τ_D exceeds the time duration of the wave packet, the coincidence rate R_c becomes constant and independent of the time delay τ_D between the wave packets. For wave packets of Gaussian shape and bandwidth $\Delta\omega$, and with a 50%:50% beam splitter, one finds that R_c is given by (see Box B)

Box B: Measuring the time separation between two photons.

The photon pair s,i from the downconverter is mixed at a symmetric 50%:50% beam splitter after a known differential time delay τ_D is introduced between the signal (s) and idler (i). The joint probability for detecting two photons at the two beam-splitter outputs 1 and 2 at time t and $t+\tau$ is proportional to

$$P_{12}(t,t+\tau)=\langle \hat{E}_1^{(-)}(t)\hat{E}_2^{(-)}(t+\tau)\hat{E}_2^{(+)}(t+\tau)\hat{E}_1^{(+)}(t)\rangle.$$

and when this is evaluated, the result is expressible in the form.

$$P_{12}(t,t+\tau)=K[g^2(\tau_D+\tau)+g^2(\tau_D-\tau)-2g(\tau_D-\tau)g(\tau_D+\tau)].$$

Here $g(\tau)$ is the Fourier transform of the spectral function that characterizes the downconverted field, and $g(\tau)$ is real, symmetric, and normalized so that $g(0)=1$. The rate $\mathcal{R}_c$ at which photon pairs in beams 1 and 2 are detected in coincidence is given by

$$\mathcal{R}_c=\int_{-T_R/2}^{T_R/2}P_{12}(t,t+\tau)d\tau,$$

where T_R is the resolving time of the coincidence detector. When T_R greatly exceeds the reciprocal bandwidth $\Delta\omega$ of the downconverted light, the limits on the integral effectively become $\pm\infty$, with the result that

$$\mathcal{R}_c=2K\int_{-\infty}^{\infty}d\tau g^2(\tau)\left\{1-\int_{-\infty}^{\infty}d\tau g(\tau_D-\tau)g(\tau_D+\tau)\bigg/\int_{-\infty}^{\infty}d\tau g^2(\tau)\right\}.$$

In the special case in which $g(\tau)$ is Gaussian and of the form

$$g(\tau)=e^{-(\tau\Delta\omega)^2/2},$$

we obtain

$$\mathcal{R}_c=(2\sqrt{\pi}/\Delta\omega)K[1-e^{-\tau_D^2(\Delta\omega)^2}].$$

$$R_c\propto K[1-e^{-\tau_D^2(\Delta\omega)^2}]. \tag{10}$$

The two-photon coincidence rate R_c is therefore expected to vary with the time delay τ_D as in Fig. 9. This has indeed been observed in an experiment in which the differential time delay τ_D was introduced artificially by translating the beam splitter BS in Fig. 8 (Hong *et al.*, 1987). It is worth noting that the measurement achieved a time resolution of a few femtoseconds, which is a million times shorter than the time resolution of the photon detectors and the associated electronics. This is possible because the measurement was really based on optical interference. In some later experiments the resolution time was even shorter than the period of the light. The same principle has been used by Chiao and co-workers to measure photon tunneling times through a barrier.

VII. CONCLUSIONS

We have seen that quantum effects can show up in both one-photon and two-photon interference. The analysis of some interference experiments confronts us with fundamental questions of interpretation and brings out that the quantum state reflects not what we know about the system, but rather what is knowable in principle. This avoids any reference to consciousness in the interpretation of the state. Finally, quite apart from their fundamental interest, quantum interference effects have led to some valuable practical applications, such as the new method for measuring the time separation between two photons on a femtosecond time scale, and new techniques of controlling the degree of coherence of two light beams without change of intensity.

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